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A note on the slope of a density interface between two stably stratified fluids under wind

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Experiments were conducted with two layers of stably stratified fluid in a wind-wave tank. The slope of the density interface was measured and was related to the wind stress, the density difference between the two fluids and the depth of the interface.

1. Introduction

A layer of lighter fluid above a heavier fluid is a common occurrence; seasonal thermoclines in oceans and in lakes and man-made heat disposals in natural water bodies are notable examples. As a result of the ever-present wind acting on the water surface, the upper layer is generally turbulent, and the density interface between the two fluids is tilted. Results are presented of an experiment on the slope of the density interface conducted in a wind-wave tank.

2. Experiment

The present results are derived from an earlier study on wind-induced entrainment at a stable density interface (Wu 1973), and the following experimental procedure and conditions have been abstracted from this study.

The experiment was conducted in a transparent wind-wave tank 20·5 cm wide and 232 cm long. A blower was installed at the upwind end of the tank and a wave absorber at the downwind end. The tank was covered for the first 196 cm to provide a 9·5 cm high wind tunnel over 28 cm deep water. As illustrated in figure 1, the tank was filled with two layers of fluid, blue-coloured fresh water lying over clear salt water of various densities. The thickening of the blue layer along with the tilting of the density interface under a steady wind were photographed with a movie camera.

The wind-velocity profiles near the water surface were measured at five stations along the tank. The friction velocities of the wind at these stations, obtained from the logarithmic wind profiles, were compiled to determine the average wind friction velocity $\overline{u}_{*a}$ in the tank. From the average friction velocity and the average roughness length (also obtained from the wind profile), the wind boundary layer in the tank was found to be aerodynamically rough for all the tests.

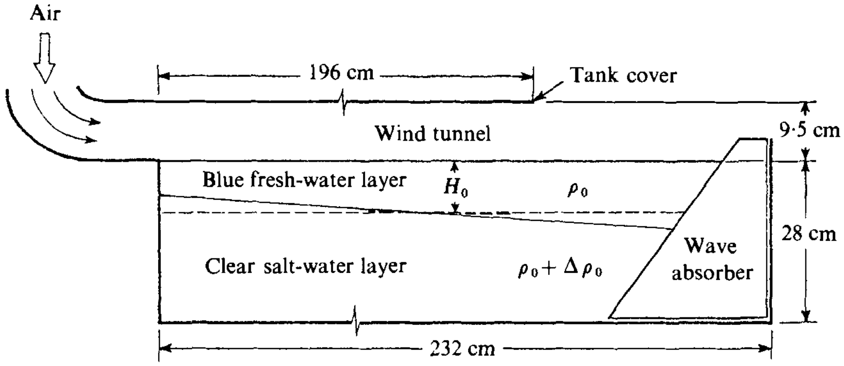

FIGURE 1. General arrangement.

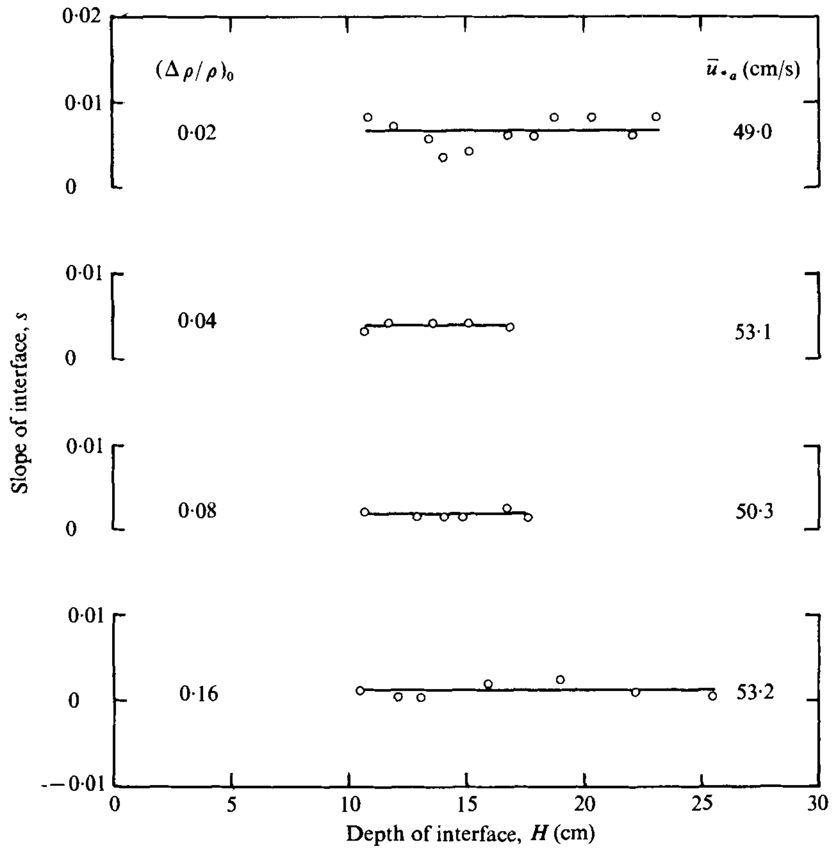

FIGURE 2. Slope of interface with various density stratifications. The initial depth of the upper layer is 10 cm.

3. Results

Slopes of density interface

During the experiment, very violent turbulent motion was clearly observed within the blue layer. The interface was wavy and rough, with high frequency disturbances superimposed on short-crested irregular waves. The blue layer, always homogeneous in

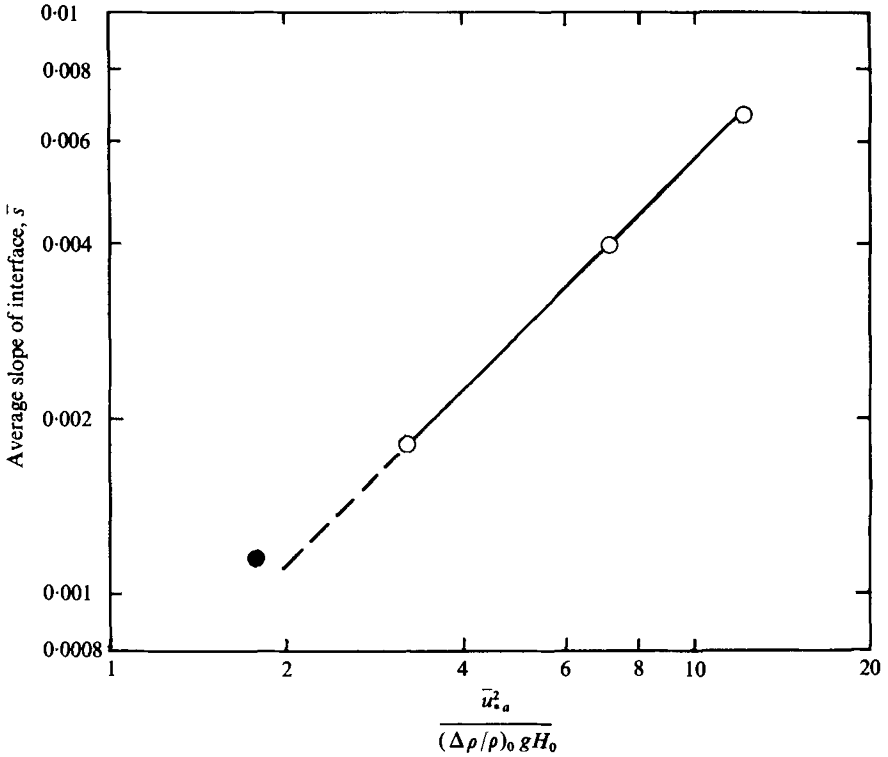

FIGURE 3. Variation of interfacial slope with Richardson number.

colour, marked the turbulent mixing zone, which had a sharp density jump at its lower boundary. The density interface was tilted, with the upwind end higher than the downwind end.

The upper and lower boundaries of the mixed (blue) layer over the middle two-thirds of the tank length were traced from the film. The trace of the lower boundary was then digitized and fitted with a straight line by the method of least squares to determine the slope of the density interface. This procedure, however, was found unsatisfactory for determining the slope of the free water surface, where long and large amplitude waves prevail. At least three traces were made from adjacent frames of the film for each depth of the interface. The average slope $\bar{s}$ of the interface and the average thickness H of the mixed layer obtained from adjacent traces are plotted in figure 2. The wind friction velocity $\bar{u}_{*a}$ and the initial non-dimensional density difference at the interface $(\Delta\rho/\rho)_0$ for each test are also indicated in the figure; $\Delta\rho$ is the density difference between the two fluids, ρ is the density of the upper fluid and the subscript 0 indicates the initial conditions before the wind started to blow. No systematic variation of the slope with the depth of the interface is indicated by the data and a line is drawn in figure 2 to indicate the overall average slope of the interface for each test.

Variation of the interfacial slope with Richardson number

Turbulence effects in stratified fluids are generally governed by the Richardson number *Ri* (Turner 1973, chap. 9), the form of which used here is

$$Ri = g\Delta\rho_0 H_0/\rho_0\bar{u}^2_{*a}, \qquad (1)$$

where g is the gravitational acceleration. As pointed out earlier (Wu 1973), in the present tank the density difference across the interface decreases as the thickness of the upper layer increases; the two quantities are inversely proportional, i.e.

$$H\Delta\rho = H_0\Delta\rho_0 = \text{constant}.$$

Consequently, the Richardson number defined in (1) has a constant value for the present mixing system involving two layers of stably stratified fluid under a steady wind. In other words, as the upper layer thickens in such a system, the weakened disturbing effect of the wind at a greater depth is counter-balanced by the simultaneously reduced stabilizing effect of a smaller density difference across the density interface. Physically, this is why the Richardson number remains unchanged during the experiment, and appears to provide the explanation for the slope of the interface being invariant with the depth of the interface during each test.

The average slope $\bar{s}$, for each test, is plotted versus the Richardson number in figure 3. Note that the slope determined at $(\Delta\rho/\rho)_0 = 0{\cdot}16$, shown as a solid circle in figure 3, is indeed small and was obtained with less accuracy. A straight line was then fitted to the other three points, shown as the open circles in figure 3, to obtain the following formula:

$$\bar{s} = 5{\cdot}4(g\Delta\rho_0 H_0/\rho_0\bar{u}_{*a}^2)^{-1}\times 10^{-4}. \tag{2}$$

4. Discussion

Effects of side walls on wind-mixing experiments

Laboratory experiments in a two-layer flowing system were conducted by, among others, Keulegan (1958, 1966). In one series of experiments he studied the motion of saline water in still fresh water, and in another series a layer of fresh water was flowing over a still layer of salt water. In both cases, the flow was shown by Keulegan to depend on the width–depth ratio of the flowing layer and on the Reynolds number based on the depth of the flowing layer. Since the shear stress at the density interface and on the side walls in these cases should be as important as the resistance on the tank bottom, Wu (1969) suggested that, instead of the depth, the hydraulic radius of the flowing layer should be used as the length parameter in the Reynolds number as this characterizes the resistance to the flow. The hydraulic radius is the ratio of the cross-sectional area to the perimeter of the flowing layer. With this suggested length parameter, Keulegan's results obtained with various width–depth ratios were shown to depend only on the Reynolds number (Wu 1969). In summary, for a two-layer flowing system the width–depth ratio is an important flow parameter, and the use of the hydraulic radius appears to encompass the effects of the width–depth ratio.

The flow conditions in the present two-layer wind-mixing system are different from those in a two-layer flowing system. In the flowing system, the mean motion is roughly uniform in the flowing layer; while in the wind-mixing system, the vertical extents of both the wind-induced drift currents near the water surface and the return currents just above the interface are very limited (Baines & Knapp 1965; Wu 1975). The maximum drift current exists at the water surface and the maximum return current near the interface. Both the drift current and the return current decrease very rapidly towards the middle of the mixed layer. There is essentially no mean flow over the major portion of the mixed layer. Consequently, the effects of the side walls are much less important in the wind-mixing system than in a flowing system. The 'effective' hydraulic radius of the mixed layer is approximately the depth of the interface. Moreover, as the interface deepens during the present experiment the vertical extents of the drift current and the bottom current do not vary appreciably with the thickness of the upper layer. Therefore, as the mixed layer deepens with a decreasing 'apparent' width–depth ratio, no significant change of the 'effective' hydraulic radius takes place. Consequently, no systematic variation with this ratio is seen in the earlier results on entrainment (Wu 1973) or in the present results on interfacial slope.

Comparison with other results

Under a steady wind, the slope s of the thermocline was proposed by Hellström (1941) to be represented by the following empirical formula:

$$s = 0{\cdot}037 a U_6^{1\cdot 8}/gH_0 \Delta\rho_0, \tag{3}$$

where U_6 is the wind velocity measured 6 m above the mean water surface, a is a constant having a value between 1·0 and 1·5; as defined here, H_0 is the initial depth and $\Delta\rho_0$ is the initial density jump at the thermocline. In this formula, the quantities in the denominator are in cgs units, while U_6 is measured in m/s.

Although it is difficult to compare Hellström's dimensional expression with the present results, it is interesting to note that, if we assume a constant wind-stress coefficient, we can rewrite (3) as

$$s \sim u_{*a}^{1\cdot 8}/gH_0 \Delta\rho_0, \tag{4}$$

in which s has nearly the same functional form as that of the present results, given in (2). If we change the exponent in (4) from 1·8 to 2 and adopt a value of 3×10^{-3} for the wind-stress coefficient, we can rewrite (4) in the same form as (2) with a proportionality constant of 12·3. This value is of the same order of magnitude as the present coefficient, 5·4. A slightly larger wind-stress coefficient has been adopted on the basis that Hellström's result was obtained at a short fetch and that he measured the wind velocity at a height of 6 m instead of 10 m, the standard anemometer height.

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REFERENCES

Baines, W. D. & Knapp, D. J. 1965 Wind driven water currents. *J. Hyd. Div. A.S.C.E.* **91**, 205–221.

Hellström, B. 1941 Wind effect on lakes and rivers. *Ingen Vetensk Akad. Handl.* **158**, 1–188.

Keulegan, G. H. 1958 Twelfth progress report on model laws for density currents – the motion of saline fronts in still water. *Nat. Bur. Stand. Rep.* no. 5831.

Keulegan, G. H. 1966 The mechanics of an arrested saline wedge. In *Estuary and Coastline Hydrodynamics* (ed. A. T. Ippen), pp. 546–574. McGraw-Hill.

Turner, J. S. 1973 *Buoyancy Effects in Fluids*. Cambridge University Press.

Wu, J. 1969 Laboratory results of salt water intrusion. *J. Hyd. Div. A.S.C.E.* **95**, 2209–2213.

Wu, J. 1973 Wind-induced turbulent entrainment across a stable density interface. *J. Fluid Mech.* **61**, 275–288.

Wu, J. 1975 Wind-induced drift currents. *J. Fluid Mech.* **68**, 49–70.